A new method for SAR images speckle reduction

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Abstract: Synthetic Aperture Radar (SAR) image is polluted easily by speckle noise, which can affect further processing of SAR image. Traditional methods employ wavelet transform, which is only effective in representing point singularities. Based on Fast Discrete Curvelet Transform (FDCT), a de-noising method for SAR image is presented. FDCT is employed to transform the SAR image into the curvelet domain to obtain the curvelet coefficients, and then soft and hard thresholding de-noising processes are performed separately on the Curvelet coefficients of different scales and directions by using adaptive threshold estimation. Finally the SAR image is reconstructed by inverse FDCT. This de-noising method is applied to the experiments of a single look SAR image, and compared with the wavelet de-noising method. Experimental results indicate that based-FDCT de-noising method is a more effective method, which is not only better in reducing speckle, but also of advantage in holding information of target edge and grain.

Key words: synthetic aperture radar, speckle, fast discrete curvelet transform (FDCT), image denoising

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1 INTRODUCTION

In synthetic aperture radar (SAR) system, the image is easily polluted more or less, which causes great trouble for further processing of the image. A number of methods have been developed in the last decade to deal with the speckle noise, such as average filtering, median filtering, order statistics filtering, and other filtering methods working in transformed domain. Among them the wavelet method (Donoho D L, 1995; Chang S G, 2000) is quite mature.

Wavelet transform is optimal only when representing one dimensional piecewise smooth signal with point singularities, and losses its advantage for two dimensional signal or signals with line singularities. In view of this, Candes and Donoho (2004) presented ridgelet transform which can effectively represent two dimensional signals with line singularities. Furthermore, they proposed curvelet transform and gave its implementation based on ridgelet transform so as to represent high dimensional signals with curve singularities (Candès E J & Donoho D L, 2004; Starck J L *et al.*, 2002). Curvelet transform takes edges as basic representative elements and is anisotropic with direction, which is helpful to represen edges effectively. Due to the complexity of the basis curvelet function, it is hard to implement. Curvelet develops in two generations. The first generation is based on ridgelet theories, which is a combination of a special filtering process and multiscale ridgelet transform. The implementation includes subband decomposition, smoothing and partitioning, regularization and ridgelet transform, so there are too many parameters with big redundancy, which inevitably causes blocking effects. Whereas the second generation curvelet transform employs novel framework, with implementation in frequency domain, so the parameters are reduced, and the process is easy and fast (Candès E J *et al.*, 2005).

The current researches on speckle denoising of SAR images (Ulfarsson M O, 2002; Xiao L, *et al.*, 2006) are mainly based on the first generation curvelet transform, whose denoising effect is fine when the edges are straight lines. However for SAR images, the edges are mainly curves, which invalidates the above method. In this paper, we propose to employ the second generation fast discrete curvelet transform (FDCT) in speckle denoising of SAR images; experiments show the advantages of the method.

2 FAST DISCRETE CURVELET TRANSFORM

Similar to the wavelet transform and the ridgelet transform, the curvelet transform theory belongs to the category of the sparsity theory. The idea is to compute the inner product between the signal or function and the curvelet function to realize the sparse representation of

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the signal or function. So the curvelet transform can be expressed as

$$c(j,l,k):=\langle f,\varphi_{j,l,k}\rangle \quad (1)$$

where j,l,k is the scale, direction and position parameter respectively.

We use the frequency window U to realize the frequency representation of the curvelet φ, where $\hat{\varphi}(\omega)=U(\omega)$. U_j is defined as follow:

$$U_j(r,\theta)=2^{-3j/4}W(2^{-j}r)V\left(\frac{2^{\lfloor j/2\rfloor}\theta}{2\pi}\right) \quad (2)$$

where $W(r), r\in(1/2,2)$ is radial window and $V(t), t\in[-1,1]$ is angular window. These windows will always obey the admissibility conditions. $\lfloor j/2\rfloor$ is the integral part of $j/2$. The support of U_j is a polar 'wedge' defined by the support of W and V.

In curvelet transform, we let

$$\hat{\varphi}_j(\omega)=U_j(\omega) \quad (3)$$

where $\hat{\varphi}_j(\omega), \omega=(\omega_1,\omega_2)\in R^2$ is the two dimensional Fourier transform of $\varphi_j(x)$. We define curvelets (as function of $x=(x_1,x_2)$) at scale 2^{-j}, orientation θ_l and position $x_k^{(j,l)}=R_{\theta_l}^{-1}(k_1\cdot 2^{-j},k_2\cdot 2^{-j/2})$ by

$$\varphi_{j,l,k}(x)=\varphi_j(R_{\theta_l}(x-x_k^{(j,l)})) \quad (4)$$

where $\theta_l=2\pi\cdot 2^{-\lfloor j/2\rfloor}\cdot l$, $l=0,1,\cdots$ $0\leqslant\theta_l<2\pi$, R_θ is the rotation by θ radians. A curvelet coefficient can be expressed as:

$$c(j,l,k):=\frac{1}{2\pi^2}\int\hat{f}(\omega)\,\overline{\varphi_{j,l,k}(\omega)}\,d\omega$$
$$=\frac{1}{2\pi^2}\int\hat{f}(\omega)\,U_j(R_{\theta_l}\omega)e^{i\langle x_k^{(j,l)},\omega\rangle}d\omega \quad (5)$$

One implementation of digital Curvelet transform is as follows:

(1) For a two dimensional function

$$f[t_1,t_2],\quad t_1,t_2\in[0,n] \quad (6)$$

we perform 2-D FFT, resulting in the frequency domain form

$$\hat{f}\left[n_1,n_2\right],\ n_1,n_2\in\left[-\frac{n}{2},\frac{n}{2}\right] \quad (7)$$

(2) In the frequency domain, resample $\hat{f}[n_1,n_2]$ at each pair of (j,l) (scale, angle), which results in

$$\hat{f}\left[n_1,n_2-n_1\tan\theta_l\right],\ \left(n_1,n_2\right)\in P_j \quad (8)$$

where $P_j=\{(n_1,n_2):n_{1,0}\leqslant n_1<n_{1,0}+L_{1,j}, n_{2,0}\leqslant n_2<n_{2,0}+L_{2,j}\}$, and $L_{1,j}$ is parameter regarding to 2^j, and $L_{2,j}$ regarding to $2^{j/2}$; which represent the length and width of support of the window $\tilde{U}_j[n_1,n_2]$.

(3) Multiply $\hat{f}$ after interpolation with window function $\tilde{U}_j$, we obtain

$$\tilde{f}_{j,l}\left[n_1,n_2\right]=\hat{f}\left[n_1,n_2-n_1\tan\theta_l\right]\tilde{U}_j\left[n_1,n_2\right] \quad (9)$$

(4) Perform inverse 2D FFT on $\tilde{f}_{j,l}$ to obtain the discrete curvelet coefficients set $c(j,l,k)$.

In the above process, the window function $\tilde{U}$ is the curvelet φ in frequency domain, which is implemented by W and V as $\tilde{U}=WV$. Employing band pass filter, the radial window function realizes the scale partitioning. The inner most layer is Coarse scale, which is a matrix composed by low frequency coefficients. The outer most layer is Fine scale, composing of high frequency coefficients. The remaining layers are middle to high frequency coefficients, named as Detail scale, on which the angle partitioning is realized by rotation and translation of angle window function.

3 NOISE REDUCTION OF SAR IMAGE BASED ON FDCT

3.1 Analysis of noise model

Since SAR image noise obeys mulplicative model, whereas curvelet denoising is based on Gaussian additive noise model, we need first to take logarithm of the noised SAR image, and then perform curvelet transform. This can be expressed as follows:

$$y[m,n]=x[m,n]\times n[m,n] \quad (10)$$

where $y[m,n]$ is the received SAR image, $x[m,n]$ is the original image, $n[m,n]$ is the multiplicative noise. Taking logarithm, we obtain:

$$\bar{y}[m,n]=\bar{x}[m,n]+\bar{n}[m,n] \quad (11)$$

where $\bar{y}[m,n]=\ln(|y[m,n]|)$. Performing FDCT based on USFFT to $\bar{y}[m,n]$, we can get curvelet coefficients, for which thresholding will be performed.

3.2 Thresholding

There are two kinds of thresholding for curvelet coefficients, namely, hard thresholding and soft thresholding. The hard thresholding is as follows,

$$f(\bar{y})=\begin{cases}\bar{y} & |\bar{y}|>\lambda\\ 0 & |\bar{y}|\leqslant\lambda\end{cases} \quad (12)$$

The soft thresholding is

$$f(\bar{y})=\begin{cases}\mathrm{sgn}(\bar{y})(\bar{y}-\lambda) & |\bar{y}|>\lambda\\ 0 & |\bar{y}|\leqslant\lambda\end{cases} \quad (13)$$

where $f(\bar{y})$ is curvelet coefficients after hard thresholding, i.e., the reconstructed coefficients, λ is the threshold, sgn is sign function.

3.3 Adaptive thresholding

In this study, we propose to use adaptive threshold and extend the BayesShrink method presented by Chang S G (2000) to discrete curvelet transform, i.e., determining the corresponding thresholds to curvelet coefficients under each scale:

$$\lambda=\frac{\sigma^2}{\sigma_x} \quad (14)$$

where σ^2 is the noise variance after curvelet transform, σ_x is the standard variance of curvelet coefficients matrix under a scale. Monte-Carlo method is employed to evaluate σ^2.

3.4 Noise reduction of SAR image

Based on the above analysis, the procedures of the processing are as follows: (1) take logarithm of images with noise, yielding the images containing approximate additive Guassian noise; (2) apply FDCT to obtain curvelet coefficients; (3) apply Monte-Carlo algorithm to evaluate the variance of noise in each level of curvelet coefficients; (4) determine the thresholds for each scale and orientation; (5) perform hard thresholding and soft thresholding to coefficients of every scale and orientation with the corresponding adaptive thresholds; (6) apply inverse curvelet transform to the coefficients to reconstruct the SAR images.

4 EXPERIMENTS AND ANALYSIS

In the experiment, we choose a single look SAR image of 256 × 256 for testing. Hard thresholding and soft thresholding are preformed for coefficients obtained by both curvelet transform and wavelet transform. Fig. 1 (a) is the original image with speckle noise; Fig. 1(b) and Fig. 1(c) are images obtained by wavelet hard thresholding and wavelet soft thresholding respectively; Fig. 1(d) and Fig. 1(e) are obtained by curvelet hard thresholding and curvelet soft thresholding respectively.

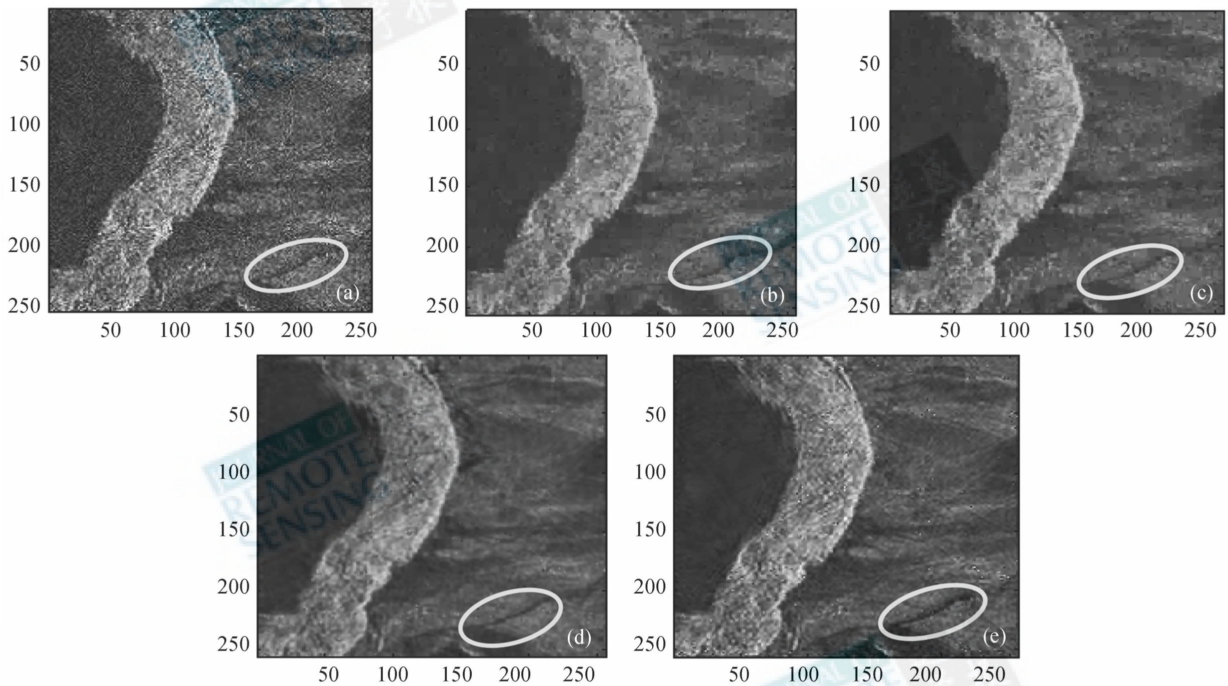

Fig. 1 Experimental comparison of several de-noising methods

(a) A single look SAR image; (b) De-noised image by hard thresholding in wavelet domain; (c) De-noised image by soft thresholding in wavelet domain; (d) De-noised image by hard thresholding in curvelet domain; (e) De-noised image by soft thresholding in curvelet domain

In order to evaluate the filtering effect of each algorithm, the following aspects are considered: (1) the ability of keeping the mean of image, directly measured by mean; (2) the deviation of the image after denoising, measured by standard deviation; (3) the noise reduction ability of filters, measured by Equivalent Number of Looks (*ENL*); (4) the denoising effect of filtering, measured by PSNR. Here, the definition of *ENL* is:

$$ENL = \begin{cases} 1/\beta^2 & \text{Brightness image of SAR} \\ (0.522/\beta)^2 & \text{Amplitudeimage of SAR} \end{cases}$$

where $\beta = \sqrt{var(\tilde{X})}/E(\tilde{X})$, $\tilde{X}$ is the gray value of filtered image, $var(\tilde{X})$ and $E(\tilde{X})$ are variance and mean.

The definition of *PSNR* is:

$$PSNR = 10\ \lg \frac{N \times \max(x_i)^2}{\sum_{i=1}^{N}[x_i - \tilde{x}_i]^2}$$

where N is the size of the original image, x_i and $\tilde{x}_i$ are the ith gray level of the original image and the filtered image.

Table 1 is the comparison of the filtering methods of Curvelet and Wavelet. We can see that all methods have good capability in keeping the mean value of images. As for the standard deviation of noise, all methods show an apparent decrease, with a decline from top to bottom, and soft threshold of Curvelet method performs best. In the aspect of speckle noise, it can be seen from the equivalent number of looks that all methods perform well, but curvelet filter works better than wavelet; and soft threshold works better than hard threshold. Concerning peak signal to noise ratio (PSNR), curvelet transform again outperforms wavelet transform. As for the edges and texture, it can be seen from the filtered images that curvelet denoising keeps the edges better than wavelet denoising does, which demonstrates that the curvelet based SAR image denoising works better than wavelet based method. In all, since curvelet transform is very sensitive to orientations, the reconstructed image appears as stripes if the noise is not fully removed in curvelet domain, whereas it appears as blocks in the wavelet case.

Table 1 Quantity comparison of several de-noising methods

	Mean	Variance	ENL	PSNR
Original	97.178	54.625	0.8594	
Wavelet hard	97.031	48.581	1.0907	21.591
Wavelet soft	97.082	48.240	1.0966	21.730
Curvelet hard	97.127	46.709	1.1791	21.895
Curvelet soft	97.164	46.515	1.1880	22.009

On retaining edge and texture information, we choose the verge points in lower right of the Fig. 1(a), and enclose them by a thick ellipse line; the same is done in Fig. 1 (b), (c), (d) and (e). According to the filtered image, Curevlet denoising is better than Wavelet in retaining edge information. Stripes can be observed in Fig. 1(d) and (e), wheras blocks can be observed in Fig. 1(b) and (c).

5 CONCLUSION

In this paper, we apply FDCT to speckle noise reduction of SAR image, and perform a comparison against wavelet method. The experiments show that the proposed not only restrains the noise with a better effect but also remains the edge and texture better.

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一种新的 SAR 图像相干斑抑制算法

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摘　要: 合成孔径雷达(SAR)图像会受到相干斑噪声的污染,对 SAR 图像的后续处理产生了很大影响。提出一种基于快速离散曲波变换(FDCT)抑制合成孔径雷达(SAR)图像相干斑噪声的方法。先通过 FDCT 把 SAR 图像变换到曲波域中,得到曲波系数,再应用自适应阈值算法估计不同尺度、不同方位曲波系数的阈值,分别对曲波系数进行硬阈值和软阈值化处理,最后通过 FDCT 反变换恢复出图像。对单视 SAR 原始图像进行处理,并与小波去噪方法进行各种量化比较,结果表明,Curvelet 滤波器要比 Wavelet 滤波器效果好,软阈值算法的效果比硬阈值算法好。基于 FDCT 的 SAR 图像相干斑去噪,不仅抑制相干斑能力比较强,而且在目标的边缘及纹理信息的保持上也有很大的优势。

关键词: 合成孔径雷达,相干斑,快速离散曲波变换(FDCT),图像去噪