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Relative height accuracy analysis of TanDEM-X system

WANG Bing-nan[1,2], **XIANG Mao-sheng**[1]

1. *The Institute of Electronic*, *Chinese Academy of Sciences*, *Beijing* 100080, *China*;
2. *Graduate School of Chinese Academy of Sciences*, *Beijing* 100039, *China*

Abstract: According to TanDEM-X satellite program, this paper studies interferometric capabilities of TanDEM-X mission during an orbit period and tries to get the change of relative height error in a global scope. Firstly, based on HILL-equation of the helix formation, effective baseline, including the range and azimuth effective baseline, which has a great impact on interferometry is analyzed in detail during an orbit period. Then the influence of baseline decorrelation imposing on interferometric phase error is discussed detailedly, which is the most important influence on relative height error. At last relative height model is established and relative height error or TanDEM-X system is presented further. The computer simulating result shows the validity of analysis.

Key words: interferometry, HELIX formation, effective baseline, correlation coefficient, phase error, relative height accuracy

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1 INTRODUCTION

TanDEM-X is a mission for spaceborne radar interferometer which is studied by both EADS Astrium team and DLR. This mission, which is planned to be completed in 2009, is based on two Terrasar-X radar satellites flying in a close formation to constitute a timely and global-scale high resolution radar interferometric system. TanDEM-X is the sister satellite of TerraSAR-X. It not only operates independently of TerraSAR-X, but also works with it synchronously. TanDEM-X system is planned to acquire surface characteristic of earth in 150 million square kilometers scale. Germany will achieve high resolution digital elevation model in global scale by 2010. This is completely new height model, which can not be restricted by the difference of region and country and also can not bring incoherence due to different ways of measurement or mosaic surface.

TanDEM-X mission is based on TerraSAR-X mission by launching a second TerraSAR-X like satellite. Both satellites fly in a close formation to achieve a flexible baseline to satisfy different applications. TanDEM-X system has no restriction on intrinsic resolution caused by time deccorelation and atmosphere interference. The main goal of the mission is the generation of global high resolution DEM corresponding to DTED/HRTI-3.

Domestic performance analysis on TanDEM-X system is few and Huang & Liang (2006) and He *et al.*, (2005) stressed on signal model, which has no all-around analysis. Krieger *et al.*, (2005) and Moreira *et al.*, (2004) compared the TanDEM-X system height resolution at different incident angle without presenting interferometric capability model. This paper studies interferometric capabilities of TanDEM-X mission during an orbit period and presents a whole flow (Fig. 1) of performance analysis of distributed spaceborne SAR. Based on satellite formation and baseline, interferometric phase error can be acquired by studying the correlation coefficient of system. Then the relative height error model can be established. At last computer simulation is used to validate the analysis. This flow is applied to general distributed SAR performance analysis.

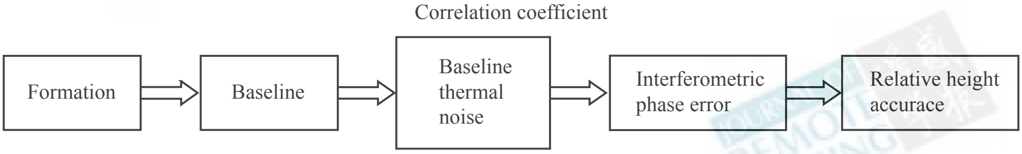

Fig. 1 Distributed spaceborne InSAR performance analysis flow

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First-author Biography: WANG Bing-nan (1984—), male. He achieved his BS degree in Measurement & Control Technologies and Instrumentation at Chinese University of Geosciences (Beijing). He is now pursuing a MSc degree in signal and information management in the Institute of Electronic, Chinese Academy of Sciences. E-mail: wangbingnan05@gmail.com.

Table 1 HRTI-3 standard

Requirement	Specification	HRTI-3
Relative Vertical Accuracy	90% linear point to point error	2m(slope < 20%); 4m(slope > 20%)
Absolute Vertical Accuracy	90% linear error	10m
Horizontal Accuracy	90% circular error	10m
Spatial Resolution	Independent pixels	12m

2 PERFORMANCE ANALYSIS RATIONALE

Relative height accuracy is decided by interferometric phase error which is related to the correlation of system. As follows TanDEM-X system performance analysis can be conducted from six aspects which are satellite formation, effective baseline, SNR analysis, correlation coefficient, interferometric phase error and relative height accuracy.

2.1 Satellite formation

Distributed small satellite formation decides the possible baselines combination, which has a great impact on system's performance. On one hand formation observes kinetic rule, on the other hand orbit can be controlled to improve the performance. Currently popular formations include interferometric cartwheel which is presented by CNES without orbit control and interferometric pendulum which is presented by DLR with orbit control. TerraSAR-X and TanDEM-X fly in a close formation which needs fuel to keep the formation steady. In order to achieve the mission there are some requests as follows:

- During interferometric imaging the two spacecraft should be separated by an effective baseline of 500—4000m.
- The along-track separation during data taking should be less than 2000m. Ideally, the along-track separation should vanish.
- Proximity operations of TerraSAR-X and TanDEM-X should be conducted to minimize the risk of a collision.

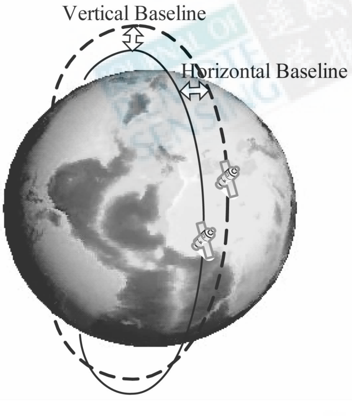

Fig. 2 HELIX satellite formation

According to several points above, HELIX satellite formation has finally been selected (Fig. 2). This formation acquires along track baseline by different eccentricity vectors and acquires across track baseline by different ascending nodes. The relative movement looks like a helix for the satellites along the orbit.

According to HILL-equation which describes the relative movement of satellites, both satellites' relative movement can be computed by

$$\begin{aligned} x &= a\Delta u + 2ae_A \sin(nt+\theta) \\ y &= -ae_A \cos(nt+\theta) \\ z &= -aj\sin(nt+\omega_1-\varphi) \end{aligned} \qquad (1)$$

with the velocity direction x, the relative position along the radius vector y, perpendicular to the orbit plane direction z, semi major axis a, eccentricity displacement e_A, initial phase θ, satellite mean velocity n, argument of latitude Δu, both orbit planes with an included angle j, argument of perigee of satellite ω_1, and $\varphi = \omega_1 - \theta$. Unknown parameters in (1) can be deduced by equation (2) and (3)

$$\cos k = \cos\varphi\cos\Delta\Omega + \sin\varphi\sin\Delta\Omega\cos i_1 \qquad (2)$$

$$j = \arcsin\left(\frac{\sin i_1 \sin\Delta\Omega}{\sin k}\right) \qquad (3)$$

Where i_1 is inclination of satellite S_1, $\Delta\Omega$ is displacement of right ascension of the ascending node. According to the mission(Krieger *et al.*,2005; Moreira *et al.*, 2004), using $a\Delta\Omega = 500$m, $\Delta u = 0$, $ae_A = 1500$m. So we can compute the relative movement of both satellites. Fig. 3 shows the relative movement and its projection on coordinate plane. From Fig. 3 we can make a conclusion that the relative movement's projection on the orbit plane is an ellipse and proportion of major axis to minor axis is 2∶1; the projection on the horizontal plane is a line while in z axis is a harmonic oscillation with an amplitude of aj. Both satellites have different displacement of right ascension of the ascending node because of different orbit inclination. Therefore fuel should be consumed to keep the formation steady.

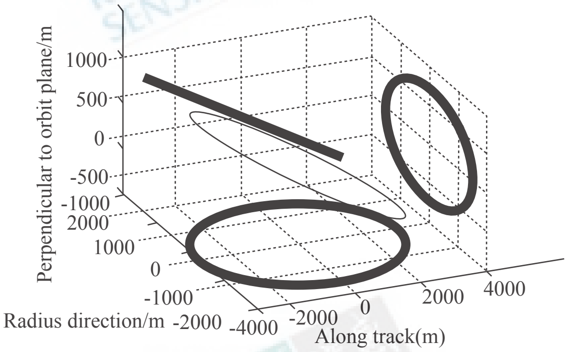

Fig. 3 Relative movement

2.2 Effective basline

Set up satellite S_1 orbit coordinate system, X_1 is velocity direction, Z_1 points to radius vector, Y_1 is perpendicular to the orbit plane. Next, we will introduce two very important effective baselines.

In Fig.4(a), **B** is baseline vector. α and β are both used to describe baseline attitude angle, where β is tilt angle of baseline vector **B** to the horizontal plane $X_1S_1Z_1$ and α is tilt angle of projection of **B** in horizontal plane to the X axis. Radar look angle is shown by θ_1 and radar look azimuth angle ψ_1 is tilt angle of look direction ST to the horizontal plane.

Fig. 4(a) shows the azimuth vertical effective baseline, which describes baseline component perpendicular to the OS_1T plane. In Fig. 4(a) draw line TS_1' which is perpendicular to the $S_1\ O$ and move coordinate system $S_1 - X_1Y_1Z_1$ to the $S'_1 - X_1'Y_1'Z_1'$, then projection of S_2 in $S_1X_1Y_1$ and $S_1'X'_1Y_1'$ are S_2' and S_2''. Again draw line $S_2''O'$ (overstriking) which is perpendicular to TS_1', well then $S_2''O'$ is perpendicular to the $OS_1\ T$ plane and $S_2''O'$ is azimuth vertical effective baseline by(4)

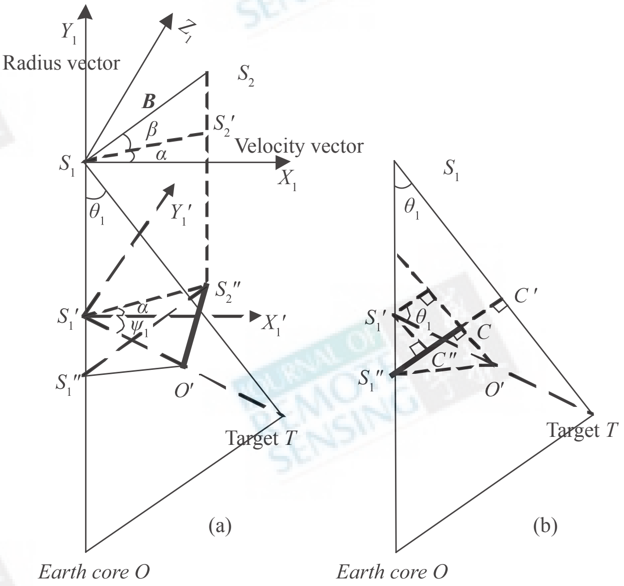

Fig.4 Effective baseline

(a) Azimuth vertical effective baseline;
(b) Range vertical effective baseline

$$\begin{aligned} \boldsymbol{B}_{\perp}^{x} &= |S_2''O'| = |S_1'S_2''| \sin(\alpha + \psi_1) \\ &= \boldsymbol{B} \cos\beta \sin(\alpha + \psi_1) \end{aligned} \tag{4}$$

Range vertical effective baseline: component which baseline vector projects to S_1OT plane projects along the direction perpendicular to radar look direction. Draw line $S_1''S_2$ which is parallel to the baseline vector **B** and project **B** to the S_1OT, then we can get vertical line $S_1''O'$. Then project $S_1''O'$ to the line $S_1''C'$ which is perpendicular to the look direction S_1T (Fig.4(b)).

$$\begin{aligned} \boldsymbol{B}_{\perp}^{r} &= |S_1''C''| + |C''C| \\ &= |S_1'S_1''| \sin\theta_1 + |S_1'O'| \cos\theta_1 \\ &= \boldsymbol{B} \sin\beta \sin\theta_1 + \boldsymbol{B} \cos\beta \cos(\alpha + \psi_1) \cos\theta_1 \\ &= \boldsymbol{B}(\sin\beta \sin\theta_1 + \cos\beta \cos\theta_1 \cos(\alpha + \psi_1)) \end{aligned} \tag{5}$$

Effective baseline is a very important parameter in interferometry. On one hand, baseline component in range affect the height accuracy because the longer the baseline is, the higher the accuracy is. On the other hand, the correlation between image pairs decreases when the baseline increases. Then interferometric phase error will go up, so designing appropriate baseline is of great importance for the interferometry.

2.3 Signal noise ratio

Before computing SNR, we should figure out the noise equivalent backscatter coefficient. Krieger *et al.* (2003) has given its formula.

$$NE\sigma^0 = \frac{\sigma^0}{S/N} = \frac{2(4\pi)^3 r_1^3 v_s k T_0 F L}{\lambda^3 G^2 \rho_r P_t \tau PRF} \tag{6}$$

$NE\sigma$ reflects the least value of backscatter coefficient of image ($S/N = 1$). In formula (6) ρ_r is ground range resolution $\rho_r = \frac{c}{2B\sin\theta}$, with B signal bandwidth, c the light velocity, θ incident angle, and G antenna power gain. $G = \frac{4\pi A}{\lambda^2}$, with A antenna area. Then formula (6) turns into (7)

$$NE\sigma^0 = \frac{\sigma^0}{S/N} = \frac{4(4\pi)^3 r_1^3 v_s k T_0 B_{rg} \sin\theta F L}{\lambda^3 G^2 c P_t \tau PRF} \tag{7}$$

Besides parameters in table 2, there are other constants including Boltzmann constant $k = 1.38054 \times 10^{-23}$, light velocity $c = 3 \times 10^8$ m/s, backscatter coefficient $\sigma^0 = -15$dB, and normal temperature $T_0 = 290$K.

according to Table 2, we can get

$$[NE\sigma^0] = 10\lg NE\sigma^0 = -21.22 \text{ dB}$$

then SNR of the system can be given by

$$[S/N] = [\sigma^0] - [NE\sigma^0] = 6.22 \text{ dB}$$

Table 2 TanDEM-X parameters

Peak Tx Power(P_t)	2260W
Chirp Bandwidth(B_{rg})	150MHz
Wavelength(λ)	0.0312m
PRF	3800Hz
Pulse Duration(τ)	47us
Figure Noise(F)	4.3dB
Loss(L)	4.1dB
Incident Angle(θ)	45°
Antenna size	4.8m × 0.7m
Average Satellite velocity(vs)	7580m/s

2.4 Interferometric phase error

Rodriguez & Martin (1992) present the formula of interferometric phase error

$$\Delta\varphi_{\xi} = \frac{1}{\sqrt{2N_L}} \sqrt{(1-\gamma^2)/\gamma^2} \tag{8}$$

Where N_L is the times of mutilooks. From (8), interferometric phase error is related to decorrelation in interferometry. There are many sources of decorrelation, of which baseline and noise are main factors.

Correlation coefficient of system can be computed by

$$\gamma = \gamma_{\text{baseline}} \cdot \gamma_{\text{SNR}} \tag{9}$$

γ_{baseline} is correlation coefficient component caused by baseline and γ_{SNR} is component caused by thermal noise, which can be given by

$$\gamma_{\mathrm{SNR}} = \frac{1}{1 + SNR^{-1}} \tag{10}$$

SNR can be computed by (7), so we only need to compute the baseline correlation coefficient. Huang & Liang (2006) and He *et al.* (2005) have given the formula.

$$\gamma_{\mathrm{baseline}} = \left(1 - \frac{B_{\perp}^{r}}{B_{\perp c}^{r}}\right) \times \left(1 - \frac{B_{\perp}^{x}}{B_{\perp c}^{x}}\right) \tag{11}$$

Where $B_{\perp c}^{r}$ is range critical baseline, $B_{\perp c}^{x}$ is azimuth critical baseline, $B_{\perp}^{r}$ is the range vertical effective baseline and $B_{\perp}^{x}$ is the azimuth vertical effective baseline. Critical baseline is the baseline when correlation coefficient is zero.

$B_{\perp c}^{r} = \dfrac{\lambda r_1}{\rho_y \cos(\theta - s)}$, where ρ_y is ground range resolution, r_1 is the distance from the target to the antenna phase center, θ is the radar look angle and s is the slope angle.

$B_{\perp c}^{x} = \dfrac{\lambda r_1}{\rho_x \sin \psi}$, where ρ_x is azimuth resolution, ψ is the radar look azimuth angle.

2.5 Relative height accuracy

Error pass coefficient of interferometric phase is deduced by He *et al.* (2005)

$$\frac{\partial h}{\partial \Delta\phi} = \frac{-\lambda H_0 r_1 \sin \theta_1}{2\pi h_0 B_{\perp}^{r}} \tag{12}$$

Where H_0 and h_0 are distances from satellite and target to the earth's core. As the small piece of region, except phase error $\Delta\phi_{\xi}$, the other errors are common ones, so relative height error can be computed by

$$h_{\xi} = \frac{\partial h}{\partial \Delta\phi} \Delta\phi_{\xi} \tag{13}$$

3 TanDEM-X PERFORMANCE SIMULATION

Satellite orbit elements and spaceborne SAR parameters are shown in Table 3.

Table 3 TerraSAR-X parameters

Satellite orbit elements		Radar parameters	
Semi major axis	6883.51km	Wavelength	0.0312m
Eccentricity	0.0011	Chirp bandwidth	150MHz
Inclination	97.4438°	Antenna Size	4.8m×0.7m
Right Ascension	88.617°	Incident Angle	45°
Argument perigee	90°	Slope Angle	20%

According to table 2 and 3, as well as formula (4) and (5), variation of effective baseline during a period is shown in Fig. 5. we can see from Fig. 5 that range effective baseline, which affects height accuracy directly, could reach 1000m, satisfying the requirement of interferometry. Range and azimuth baselines vary a bit great, which is not good for the interferometric capabilities.

As the measurement of height, range effective baseline concerns height accuracy directly. It influences not only the phase error, but also the error pass coefficient of height error to the phase. In order to gain steady interferometric capabilities, we often make it a bit short. Azimuth baseline has a impact on the correlation of system.

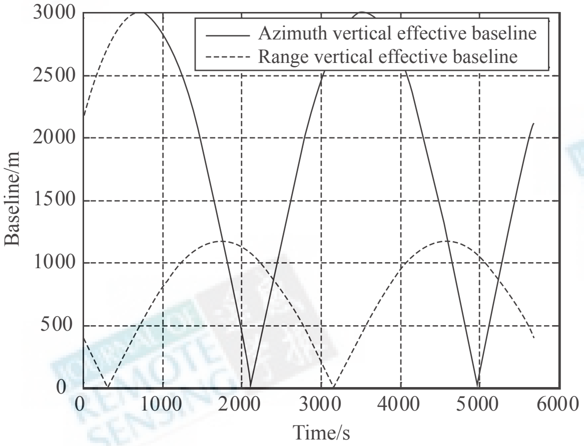

Fig. 5 Effective baseline

From fomula (9) we can gain correlation coefficient of system during a period and (8) can compute the phase error. In Fig. 6, due to unsteady of baseline during a period, correlation coefficient changes with baseline. TanDEM-X system adopts 16 looks process, which reduce the phase error in some degree. We can see from the figure that phase error is less than 10 degree, which satisfies the requirement of measurement.

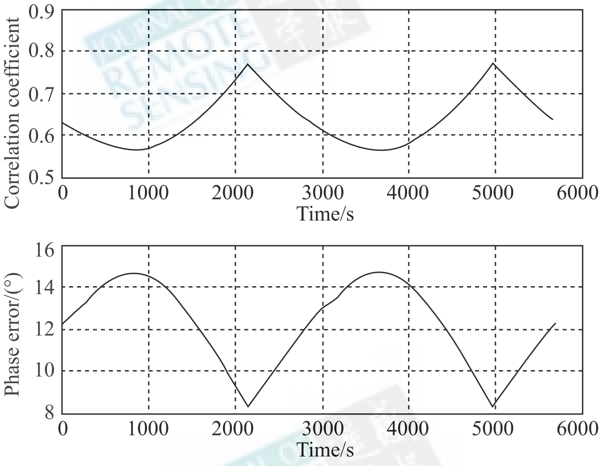

Fig. 6 Correlation coefficient and phase error

Phase error pass coefficient can be computed by fomula (12). Fig. 7 shows the variation of relative height error during a period (assigning the slope angle as 20°). There are two peaks in the figure, which is in the high latitude. That's because the range effective baseline reaches the lowest above the polar region. Between the peaks there are flat areas, which have better height accuracy, satisfying the requirement of mission and HRTI-3 standard.

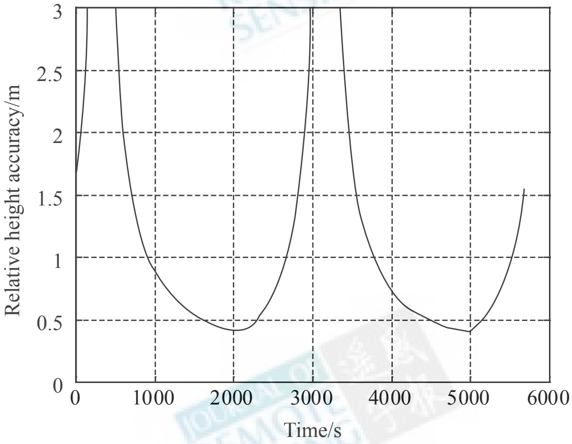

Fig. 7 Relative height accuracy

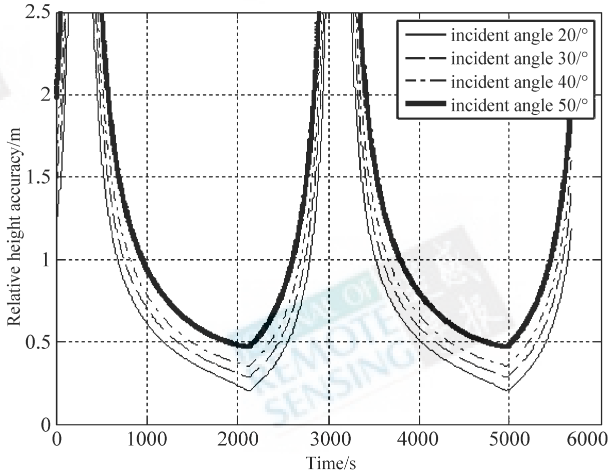

Fig. 8 Variation of height accuracy at different incident angle

Fig. 8 makes a comparison for the relative accuracy at different incident angle. The larger the incident angle is, the smaller the height accuracy becomes. This proves the result of Krieger *et al.* (2005). One of the important factors is that the signal noise ratio becomes lower when the incident angle increases. Specially, when the incident angle equals 45°, the smallest resolution given by Krieger *et al.* (2005) is 1.3m. The smallest height accuracy given in this paper reaches 0.5m, considering the baseline and thermal noise decorrelation.

4 CONCLUSION

This paper analyzed the performance of satellite TanDEM-X and TerraSAR-X systematically. Through the analysis of effective baseline, correlation coefficient, phase error and relative height accuracy during an orbit period, the result of signal simulation validates the analysis. In addition this paper hasn't considered phase synchronization, time synchronization, beam synchronization, baseline measurement and so on, which have influence on height accuracy.

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TanDEM-X 系统相对测高性能分析

汪丙南[1,2],向茂生[1]

1. 中国科学院电子学研究所,北京 100080;
2. 中国科学院研究生院,北京 100039

摘 要: 针对 TanDEM-X 卫星计划,重点分析其在一个轨道周期内的测高性能的变化情况。首先从建立双星 HELIX 构形的 HILL 方程出发,分析了一个轨道周期内直接影响干涉测高的有效基线,包括顺轨和交轨基线的影响,然后讨论了系统相关系数及其对干涉相位误差的影响,通过建立相对高程误差模型,进一步分析了系统相对高程精度。最后计算机仿真结果验证了分析的正确性。

关键词: 干涉,HELIX 构形,有效基线,相关系数,相位误差,相对高程精度